Y13 Electric Fields

Lesson 3: Electric Field Strength & Potential Gradient

Analysis of Coulomb's Law experiment

Let's review the work undertaken at the end of the last lesson, which you might (should) have been completed as part of this week's homework.

ACTIVITY

Testing Coulomb's law

Figure 5.5 shows an experimental arrangement for investigating Coulomb's law. The two polystyrene spheres are charged by a high-voltage supply. Sphere A is held in a fixed position and sphere B is free to move. A light bulb is used to cast a shadow of the spheres onto graph paper, so that their separation, and also the deflection of sphere B, can be measured more easily.

Table 5.1 shows the results for six different separations of the spheres' shadows. Between each set of measurements the spheres were recharged.

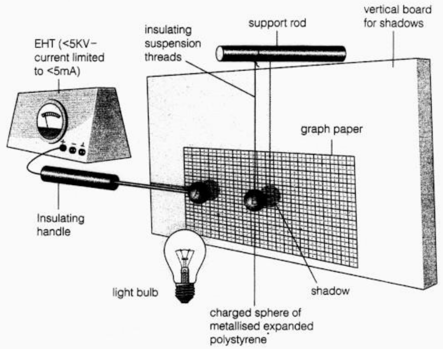
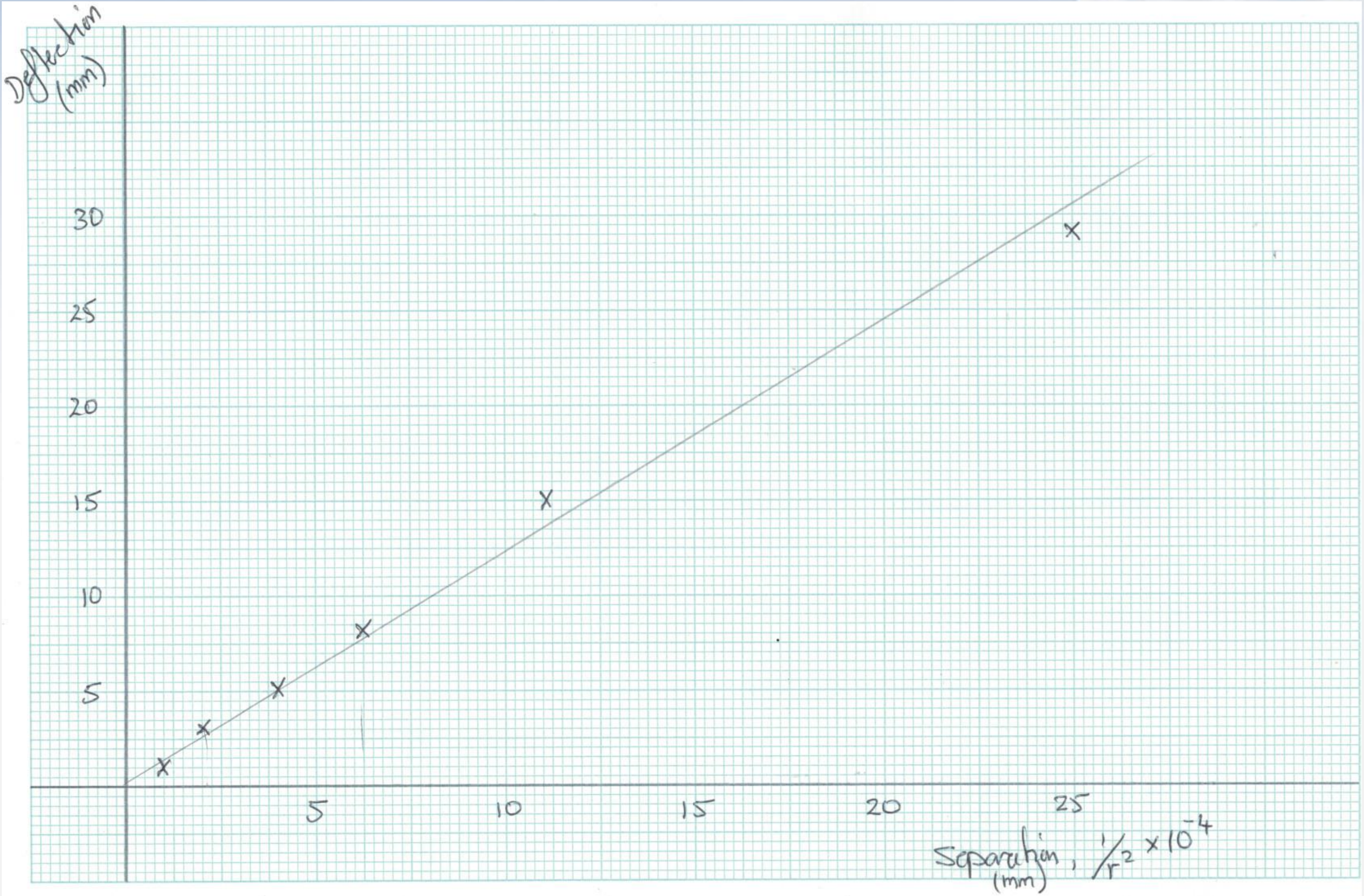


Figure 5.5

Table 5.1

Distance between centres of spheres' shadows/mm	100	70	50	40	30	20
Deflection of sphere B's shadow/mm	1	3	5	8	15	29

1 Plot a graph of the deflection of sphere B's shadow against $\frac{1}{r^2}$, where r is the separation of the centres of the spheres' shadows.
   Discuss whether or not your graph supports Coulomb's law.
2 Explain why the deflection of sphere B is proportional to the force between the balls.
3 Discuss the sources of error in this experiment. You should consider both systematic and random errors.
4 Discuss how you could calculate the real separation of the spheres, rather than their shadows.



Electric field strength

One definition of electric field strength is the force per unit charge;

$$E = \frac{F}{Q}$$

The units of electric field strength are newtons-per-coulomb.

Electric field strength is a vector: it has both magnitude and direction. The direction of the field is from positive charge to negative charge because it shows the direction of the force that would be experienced by a **positive charge** within the field.

This equation can be used for uniform fields between parallel plates and non-uniform fields due to spherical charges (these two situations will be considered separately).

Alternative definition of electric field strength

In a gravitational field, the force is the product of the gravitational field strength (g) and mass (m) giving the property that we call weight.

In an electric field, the force is the product of the electric field strength (E) and electric charge (Q) but there is no special name for the resulting force in this case!

$$F = E\,Q \qquad E = \frac{F}{Q}$$

In the case of electric fields, there is also another definition (equation) that can be used…

Thinking about the divided components in the second equation, we can treat force as energy / distance (from the definition of work done) and charge as energy / voltage (from the definition of voltage). These substitutions give;

$$E = \frac{\text{energy / distance}}{\text{energy / voltage}}$$

Therefore; $E = \dfrac{V}{d}$ (we will return to this shortly)

Radial field strength using Coulomb's Law

The third way to calculate electric field is by substituting Coulomb's Law into the ratio of force-per-unit-charge…

$$E = \frac{F}{Q} \quad \text{where} \quad F = \frac{Q_1 Q_2}{4 \pi \varepsilon_0 r^2} \quad \text{giving} \quad E = \frac{Q}{4 \pi \varepsilon_0 r^2}$$

The units of the electric field strength are still the same ($N\ C^{-1}$) but it can now be calculated using the individual components that contribute to the force, without having to know the force itself.

Only one value of charge is required because this is the electric field strength around that charge at different distances. But a second charge is implied! The field strength calculated is the effect (force per unit charge) that would be experienced by a **positive unit test charge** that is inserted into the field.

Potential and potential gradients

When an object is held in a specific position within a field, it has a quantity of electric potential energy that can be calculated. The change in potential energy with position is the potential gradient, which shows the direction in which work is done within a field.

To understand this, it is best to think first about the familiar gravitational field;

- The gravitational field acts "downwards" towards the centre of the Earth.
- To do work against gravity, an object must be moved "upwards", storing gravitational potential energy by an amount that depends on the gravitational field strength and the height moved (increase in distance from the source of the gravitational field).

In the case of gravitational potential energy, the work done is weight x height, which is simply a specific version of the familiar equation; work done = force x distance

Electric fields behave in a very similar way.

Electric potential in radial fields

The electric potential energy is defined in joules-per-coulomb (voltage) and is related to the electric field through the "alternative" equation for field strength.

$$E = \frac{V}{d}$$ rearranges to give $$V = E\,d$$

We then substitute using the full form of the electric field equation;

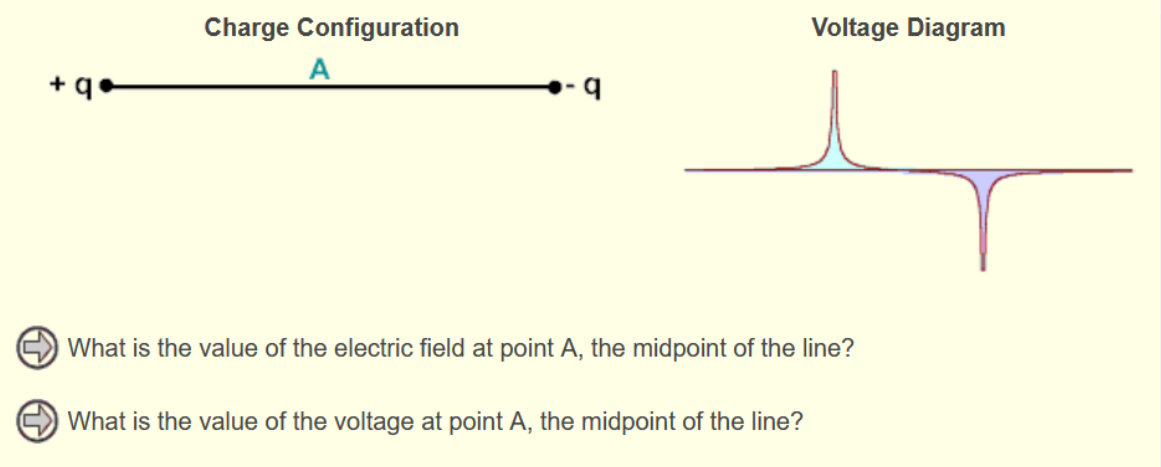
$$E = \frac{Q}{4 \pi \varepsilon_0 r^2}$$ to get $$V = \frac{Q}{4 \pi \varepsilon_0 r}$$

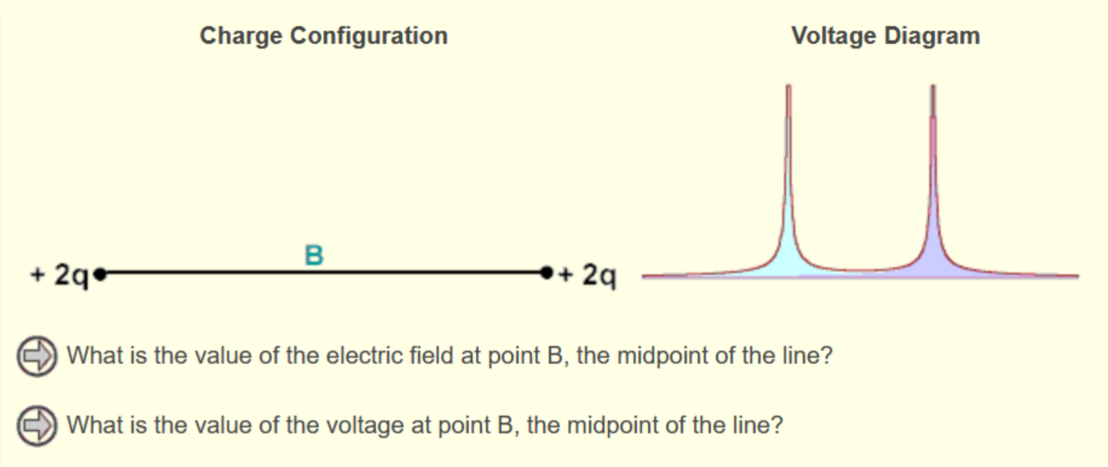
The electric potential gives the voltage (energy per unit charge) at a certain distance (r) from the centre of the originating charge, Q. It follows an inverse (not inverse square) relationship. The electric potential has positive values for positive originating charges and negative values for negative originating charges.

Electric potential and electric field

All charged spheres generate radial fields and those fields have associated potentials. The direction of the field depends on the polarity of the originating charge but in all (spherical) cases the fields gradually decrease all the way "to infinity", varying as $1/r^2$.

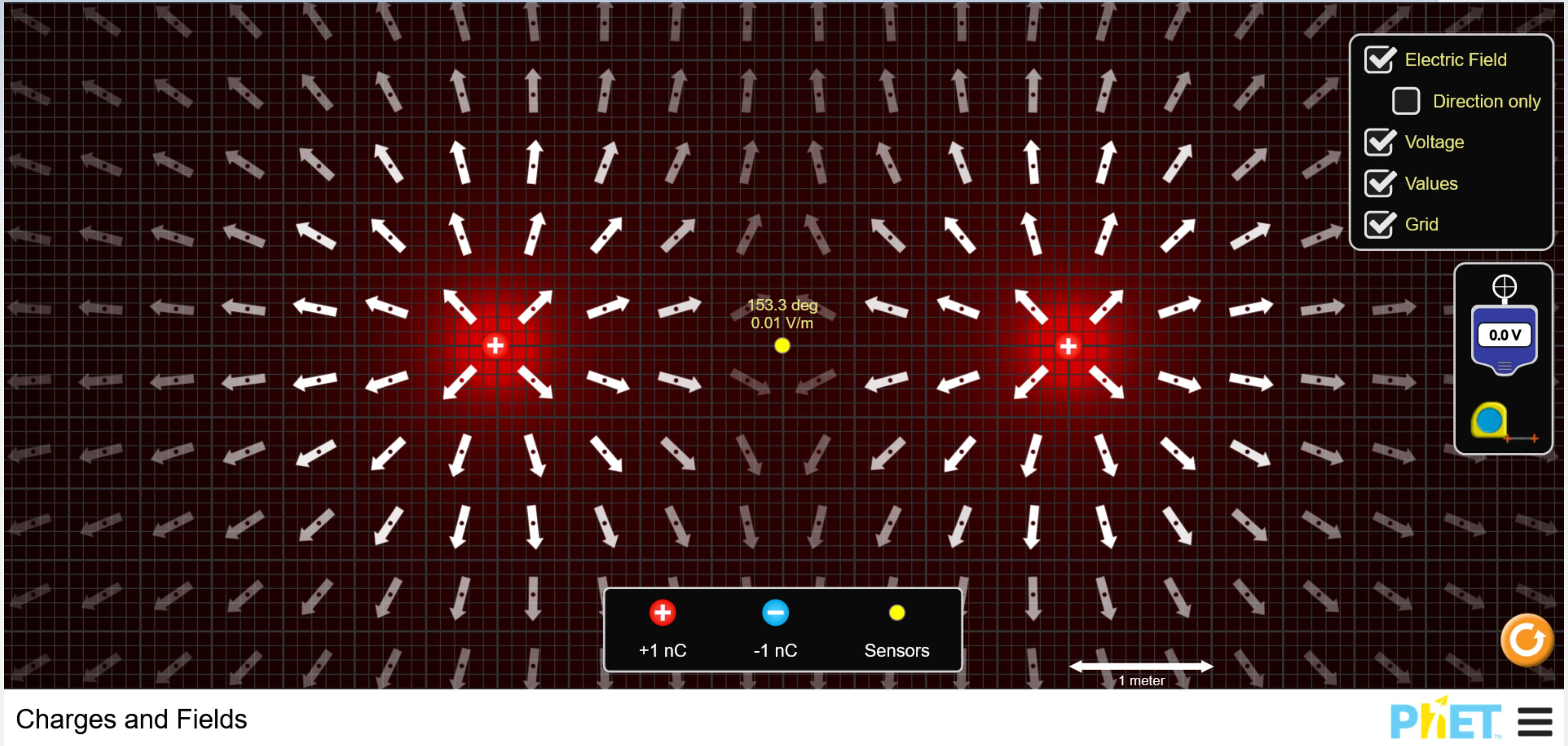
If there are multiple charged spheres, there are locations where the resultant field is zero or the resultant potential is zero: occasionally, both are zero in the same location.





Assume the spheres are separated by 2r. Source: http://dev.physicslab.org/Document.aspx?doctype=3&filename=Electrostatics_PointChargesVoltage.xml

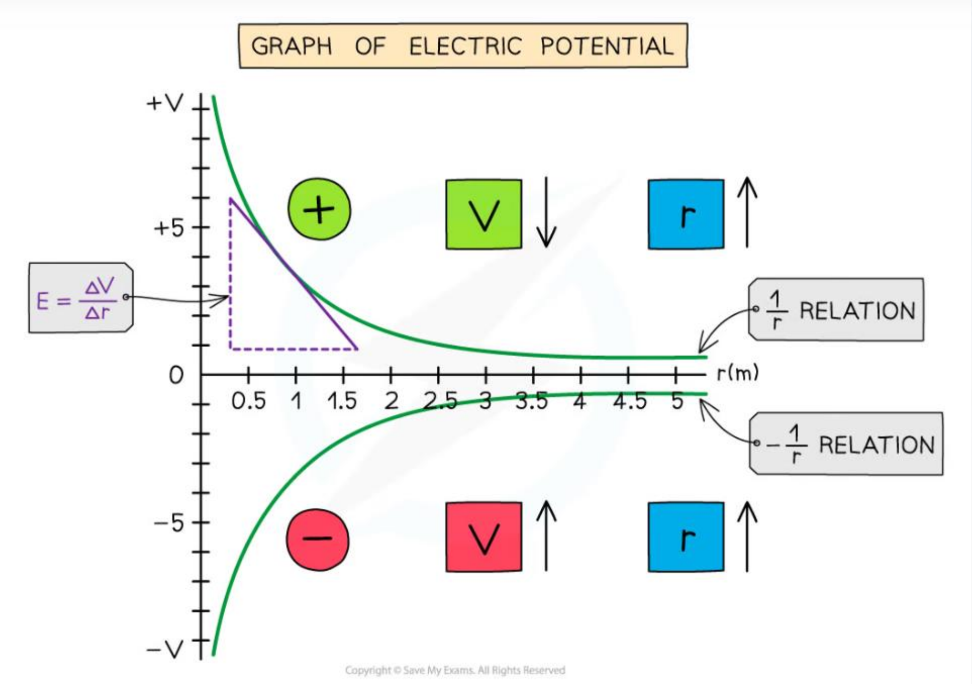
Exploring fields and potentials



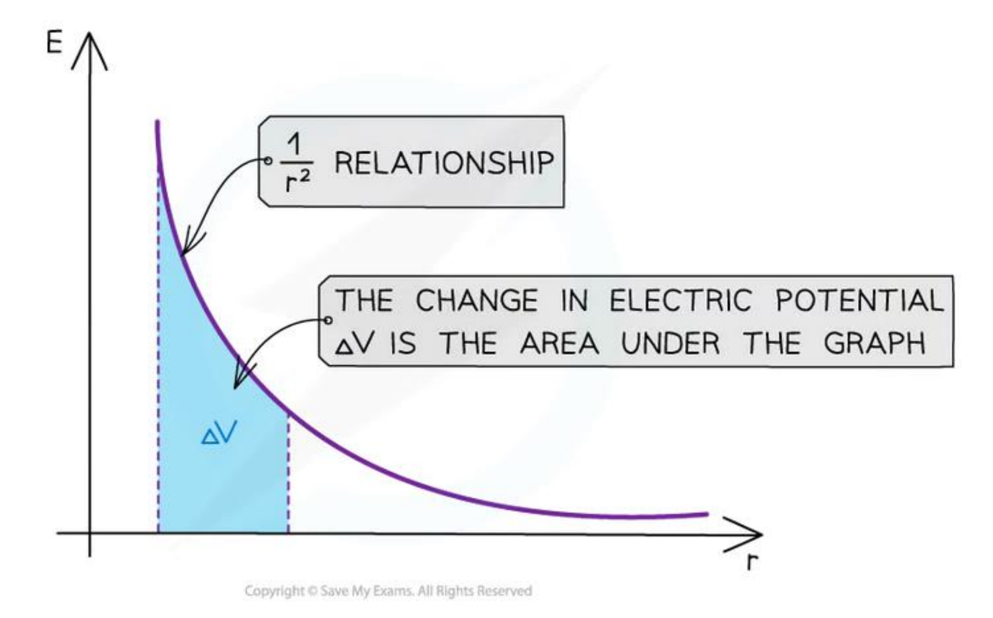
Source: https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_all.html

Electric field **is** potential gradient

The electric potential gradient (field) is the rate of change of electric potential with distance from the originating charge. The gradient is calculated using the tangent of the curve in a graph of electric potential versus distance (see right).



$$E = \frac{\Delta V}{\Delta d}$$



Similarly, in a graph of electric field (potential gradient) versus distance, the change in electric potential is given by the area under the graph, bounded by the line and the x-axis (see left).

Source: https://www.savemyexams.com/a-level/physics/aqa/17/revision-notes/7-fields--their-consequences/7-5-electric-potential/7-5-2-graphical-representation-of-electric-potential/

Work done in an electric field

The work done when a charge travels through an electric field is force x distance moved.

The force in this case can be taken from the definition of the electric field; $E = F / Q$

Therefore, work done = $E\, Q\, \Delta d$ (where Δd represents the change in position in the field)

But E is also defined as the potential gradient, $E = \Delta V / \Delta d$, so we can do a substitution;

work done = $(\Delta V / \Delta d)\, Q\, \Delta d$

$$\Delta W = Q\, \Delta V$$ (the charge does not change)

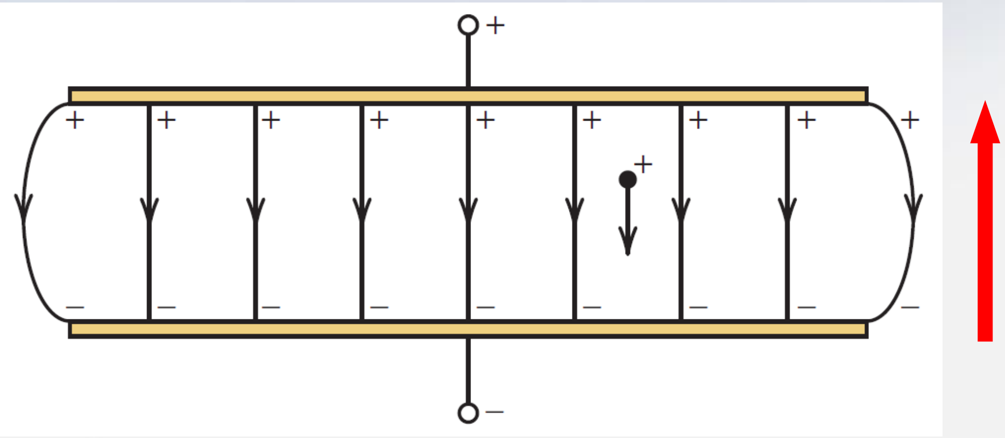
Clearly, work is done only when the charge moves through a change in potential, along a potential gradient. Trajectories (lines or surfaces) that have zero change in potential are known as equipotentials. The value of the potential is not zero, only the change in potential is zero!

If the charge moves down a potential gradient, energy is transferred away from the charge and the charge loses energy: if the charge moves up a potential gradient then work is done on the charge, which gains electric potential energy.

Electric potential in uniform fields

Whereas radial fields have just a "starting point", uniform electric fields, such as those between parallel-plate conductors, have two end-points (planes).

The electric field lines show the direction of force on a positive charge, so start on the positive plate and finish on the negative plate.



The electric potential is in the opposite direction: it starts from zero, on the negative plate, and increases towards the positive plate.

The potential difference between two points in a uniform electric field is given by;

$\Delta V = E \Delta d$ where d is the separation measured parallel to the electric field.

Once again, for separations that are entirely perpendicular to the electric field, the potential difference is zero and the movement is along an equipotential.